

CALCULUS

Exercise Set 7

Multiple Integrals

1 Double Integrals

1. Evaluate the integral of $f(x, y) = x^2 + y^2$ over the following regions
 - (a) The square enclosed by the vertical lines $x = -1$, $x = 1$ and the horizontal lines $y = -1$, $y = 1$.
 - (b) The polygon enclosed by the lines $y = 0$, $y = 1$, $x = -1$ and $x + y = 1$.
 - (c) The polygon whose vertices are the points $(-1, 0)$, $(0, 0)$, $(1, 1)$ and $(0, 1)$.
 - (d) The bounded region of the first quadrant determined by the parabolas $y = x^2$ and $y^2 = x$.
 - (e) The bounded region limited by the parabola $y = x^2 - 1$ and the line $x + y = 1$.
 - (f) The bounded region determined by the parabolas $y = x^2 - 1$ and $y = 1 - x^2$.
2. Repeat the above exercise with the functions
 - (a) $f(x, y) = x^3y$.
 - (b) $f(x, y) = x^3 - y^2$.
 - (c) $f(x, y) = \cos(x + y)$.
 - (d) $f(x, y) = e^{xy}$.

(e) $f(x, y) = \sqrt{4 + x^2}$.

3. Find the volume of the cylinder $x^2 + y^2 = 1$ below the plane $z = 2x + y + 3$ and above the plane $z = 0$.
4. Find the volume of the solid in the first octant enclosed by the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.
5. Find the average height of the paraboloid $z = 4 - x^2 - y^2$ over the region $0 \leq x \leq 1, 0 \leq y \leq 1$.

2 Double Integrals in Polar Coordinates

1. Integrate the function $f(x, y)$ over the region R
 - (a) $f(x, y) = x^2 + y^2 - xy - x - y$; $R = \{(x, y) / x \geq 0, y \geq 0 \text{ and } x + y \leq 3\}$.
 - (b) $f(x, y) = e^{xy} \sin y$; $R = \{(x, y) / |x| \leq 1 \text{ and } |y| \leq 1\}$.
 - (c) $f(x, y) = x^2 + y^2 - xy$; $R = \{(x, y) / x^2 + y^2 \leq 2\}$.
 - (d) $f(x, y) = x^2 + y^2 - 2xy$; $R = \{(x, y) / y \geq 0 \text{ and } x^2 + y^2 \leq 2x\}$.
 - (e) $f(x, y) = (x^2 + y^2) \ln(x + y)$; $R = \{(x, y) / 2 \leq x^2 + y^2 \leq 4\}$.