CALCULUS
Lab Practice Set 7

1 New Maple Functions

- **implicitplot3d**($equation, x = a..b, y = c..d, z = p..q$) Plots the surface defined implicitly by an equation containing variables x, y and z.
- **tubeplot**($[x(t), y(t), z(t)], t = a..b, radius = r(t)$) Plots a tubular surface along the curve $x(t), y(t), z(t), a \leq t \leq b$ with circular cross sections of radius $r(t)$
- **plot3d**($expr, x = a..b, y = c..d$) Plots the surface defined explicitly by an expression of two variables x and y.
- **plot3d**($f, a..b, c..d$) Plots the graph of a function of two variables.
- **plot3d**($\{expr1, expr2, expr3\}, x = a..b, y = c..d$) Multiplot containing three surfaces defined by two variable expressions.
- **plot3d**($\{f, g, h\}, a..b, c..d$) Multiplot containing the graph of three functions of two variables.
- **contourplot**($expr, x = a..b, y = c..d$) Level curves of the surface $z = expr(x, y)$

2 Exercises

2.1 Surfaces

1. *Cylinders* Graph the following surfaces in 3-D.

   (a) $x^2 + y^2 = 5$

   (b) $y = x^2$

   (c) $x^3 - x - z = 0$

   (d) $y^2 - z^2 = 4$

   (e) $x^3 + y^3 = 1$

2. Graph the following sets of surfaces in a 3-D multiplot:

   (a) Three circular cylinders of radius 1 each one made of lines parallel to each of the three coordinate axis.

   (b) A sphere of radius 4 and a circular cylinder of radius 1 made of lines parallel to one of the sphere diameters.

   (c) A sphere of radius 1 and two parallel planes tangent to that sphere.

3. Plot a graph of the following quadric surfaces and examine all their sections parallel to the coordinate planes.

   (a) *Ellipsoid* $3x^2 + y^2 + 2z^2 = 1$

   (b) *Hyperboloid of one sheet* $3x^2 + y^2 - 2z^2 = 1$

   (c) *Hyperboloid of two sheets* $3x^2 + y^2 - 2z^2 = -1$

   (d) *Quadric cone* $3x^2 + y^2 - 2z^2 = 0$

   (e) *Elliptic paraboloid* $3x^2 + y^2 - 2z = 0$

   (f) *Hyperbolic paraboloid* $3x^2 - y^2 - 2z = 0$

4. Plot a graph of the *Scherk's minimal surface*

$$e^z \cos x = \cos y.$$

5. Plot the graph of the surface obtained by rotating the plane curve

$$y = 3x, \qquad 0 \le x \le 5$$

around the $OX-$axis. Repeat for:

   (a) $y = \sqrt{x}, \quad 0 \le x \le 4$
   (b) $y = e^{-x^2}, \quad -10 \le x \le 10$
   (c) $y = x^2, \quad 0 \le x \le 5$
   (d) $y = \sin 2x, \quad 0 \le x \le 2\pi$

6. Plot a graph of the following tubular surfaces

   (a) A torus whose axis is a circle of radius 5 and its cross sections are circles of radius 1.
   (b) Two tori of the above dimensions positioned as two consecutive links of a chain.
   (c) A tube surface whose axis is the helix

   $$x = 3\cos 2t, y = 3\sin 2t, z = 5t, \qquad 0 \le t \le 2\pi$$

   and its cross sections are circles of radius 1.
   (d) A tube surface whose axis is the above helix and its cross sections are circles of radius $0.5 + 0.3\cos(2t)$
   (e) A tube surface whose axis is the curve

   $$x = t\cos 2t, y = t\sin 2t, z = 5t, \qquad 0 \le t \le 2\pi$$

   and its cross sections are circles of radius $t^2/10$.

2.2 Two variable functions

1. Let $f(x, y) = 4 - x^2 - y^2$.

   (a) Plot a graph of $z = f(x, y)$ on the domain

   $$D = \{(x, y) \in \mathcal{R}^2 / -1 \le x \le 1, -1 \le y \le 1\}.$$

(b) Draw a multiplot of f and $z = 0$ on the domain D.

(c) Repeat the above questions for the following domains:

i. $D = \{(x, y) \in \mathcal{R}^2 / x^2 + y^2 \leq 1\}$.

ii. $D = \{(x, y) \in \mathcal{R}^2 / x^2 + y^2 \leq 1,\ 0 \leq x,\ 0 \leq y\}$.

iii. $D = A - B$ with $A = \{(x, y) \in \mathcal{R}^2 / x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathcal{R}^2 / 0 \leq x,\ 0 \leq y\}$.

iv. $D = A - B$ with $A = \{(x, y) \in \mathcal{R}^2 / x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathcal{R}^2 / 0 \leq x,\ 0 \leq y \leq 3x\}$.

v. $D = \{(x, y) \in \mathcal{R}^2 / x^2 + y^2 \leq 4\}$.

2. In the following cases plot a graph of the function $z = f(x, y)$ on the domain

$$D = \{(x, y) \in \mathcal{R}^2 / x^2 + y^2 \leq 1\}.$$

Experiment changing the domain.

(a) $f(x, y) = x^2 + y^2$

(b) $f(x, y) = 16(x^2 + y^2) + \dfrac{1}{x^2 + y^2}$

(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$

(d) $f(x, y) = \dfrac{\sin 3(x^2 + y^2)}{x^2 + y^2}$

(e) $f(x, y) = 2(x^2 + y^2) - 3(x^2 + y^2)^{3/2}$

3. In the following cases plot a graph of the function $z = f(x, y)$ on the domain

$$D = \{(x, y) \in \mathcal{R}^2 / -1 \leq x \leq 1,\ -1 \leq y \leq 1\}.$$

Experiment changing the domain.

(a) $f(x, y) = xy$

(b) $f(x, y) = x^2 - y^2$

(c) $f(x, y) = x(3y^2 - x^2)$

(d) $f(x, y) = \sin(2\pi x)\cos(2\pi y)$

(e) $f(x,y) = \cos(xy)$

4. Let $f(x,y) = x^2 - x^4 - y^2$.

   (a) Plot the level curves of the surface $z = f(x,y)$.

   (b) Looking at these level curves try to imagine the form of the surface. Where does the surface change most rapidly?

   (c) Plot a graph of the surface with its contour lines and check your conjectures.

   (d) Repeat the above questions for the following functions:

      i. $f(x,y) = \dfrac{||x| - |y||}{2} - \dfrac{(|x| - |y|)}{2}$

      ii. $f(x,y) = \dfrac{1}{x^2 + y^2}$

      iii. $f(x,y) = \dfrac{xye^{-xy}}{x^2 + y^2}$

      iv. $f(x,y) = (\sin x)(\sin y)e^{-\sqrt{x^2+y^2}}$

5. Let

$$f(x,y) = 100 - 20x^2 - 30y^2.$$

   (a) Calculate the partial derivatives f_x and f_y at the point $P = (1,1)$

   (b) Draw a multiplot with the following elements:

      i. The surface $z = f(x,y)$.

      ii. The curve where the plane $y = 1$ intersects the above surface.

      iii. The line of the plane $y = 1$ that passes through the point $(1, 1, f(1,1))$ and whose slope is $f_x(1,1)$.

   (c) Draw a multiplot with the following elements:

      i. The surface $z = f(x,y)$.

      ii. The curve where the plane $x = 1$ intersects the above surface.

      iii. The line of the plane $x = 1$ that passes through the point $(1, 1, f(1,1))$ and whose slope is $f_y(1,1)$.

   (d) Repeat in the following cases:

      i. $f(x,y) = x^2 - xy + y^2$ and $P = (1,2)$.

ii. $f(x,y) = e^{-(x^2+y^2)}$ and $P = (1/2, 1/3)$.
iii. $f(x,y) = (x^3 + y/2)^{2/3}$ and $P = (1,1)$.
iv. $f(x,y) = x^2y$ and $P = (1,-1)$.

6. Let

$$f(x,y) = 100 - 20x^2 - 30y^2.$$

(a) Find the linearization $L(x,y)$ of $f(x,y)$ at the point $P = (1,1)$

(b) Draw a multiplot of the surface $z = f(x,y)$ and the plane $z = L(x,y)$.

(c) Repeat in the following cases:

i. $f(x,y) = x^2 - xy + y^2$ and $P = (1,2)$.
ii. $f(x,y) = e^{-(x^2+y^2)}$ and $P = (1/2, 1/3)$.
iii. $f(x,y) = (x^3 + y/2)^{2/3}$ and $P = (1,1)$.
iv. $f(x,y) = x^2y$ and $P = (1,-1)$.

7. Let

$$f(x,y) = 100 - 20x^2 - 30y^2.$$

(a) Calculate the directional derivative $D_{\mathbf{u}}f$ of f at $(a,b) = (1,1)$ in the direction $\mathbf{u} = (\cos\theta, \sin\theta)$ for $\theta = 0$, $\pi/4$, $\pi/2$ and $3\pi/4$.

(b) Draw a multiplot with the following elements:

i. The surface $z = f(x,y)$.
ii. The curve of intersection of the surface $z = f(x,y)$ and the plane that contains both the $z-$axis and the line that passes through (a,b) with direction vector $\mathbf{u}$, i.e. the line

$$x = a + t\cos\theta,\ y = b + t\sin\theta,\ z = 0$$

for each value of θ given above.
iii. The tangent lines to these curves at the point $(a, b, f(a,b))$.(Observe that these tangent lines lie on the tangent plane to the surface at that point).

(c) Repeat in the following cases:

i. $f(x,y) = x^2 - xy + y^2$ and $(a,b) = (1,2)$.

ii. $f(x,y) = e^{-(x^2+y^2)}$ and $(a,b) = (1/2, 1/3)$.
iii. $f(x,y) = (x^3 + y/2)^{2/3}$ and $(a,b) = (1,1)$.
iv. $f(x,y) = x^2 y$ and $(a,b) = (1,-1)$.