CALCULUS
Lab Practice Set 4

1 New Maple Commands

- **int**$(f(x), x)$ definite integral of the expression $f(x)$ with respect to x
- **Int**$(f(x), x)$ symbol of the definite integral of the expression $f(x)$ with respect to x
- **int**$(f(x), x = a..b)$ definite integral of the expression $f(x)$ on the interval $[a, b]$
- **Int**$(f(x), x = a..b)$ symbol of definite integral of the expression $f(x)$ on the interval $[a, b]$
- **middlesum**$(f(x), x = a..b, n)$ Middle Riemann Sum for a partition with n subintervals
- **middlebox**$(f(x), x = a..b, n)$ Plots the rectangles for a Middle Sum with n elements
- **leftsum**$(f(x), x = a..b, n)$ Left Riemann Sum for a partition with n subintervals
- **leftbox**$(f(x), x = a..b, n)$ Plots the rectangles for a Left Sum with n elements
- **rightsum**$(f(x), x = a..b, n)$ Right Riemann Sum for a partition with n subintervals
- **rightbox**$(f(x), x = a..b, n)$ Plots the rectangles for a Right Sum with n elements

- **for** i **from** n **to** m **by** k **do** ... **od** Repeats the instructions given in ... for values of i from n to m by steps of size k
- **assume**(k,*integer*) Limits k to integer values

2 Exercises

1. Consider the region enclosed by the graph of the function

$$f(x) = x^2 + 1.$$

   and the segment of the OX -axis between $a = 0$ and $b = 1$.

   (a) Dividing the OX -axis segment into $n = 5$ subintervals of the same length, estimate the area of the given region using rectangles whose base is each one of these subintervals and their height is the value of $f(x)$ at the mid point of the corresponding subinterval.

   (b) Repeat for $n =$ 10, 20,. . . ,50 rectangles.

   (c) Find a general expression to estimate the area using n rectangles.

   (d) Find the limit of the above expression when n approaches $+\infty$.

   (e) Using your computer algebra system, check that the definite integral of $f(x)$ on the interval $[a, b]$ is just the above limit.

2. Repeat the above exercise with rectangles whose height is the value of $f(x)$ at the left end of the subintervals.

3. Repeat the above exercise with rectangles whose height is the value of $f(x)$ at the right end of the subintervals.

4. Repeat exercises 1, 2 and 3 for the following functions

   (a) $f(x) = x^3$

   (b) $f(x) = \sin(\pi x)$

   (c) $f(x) = \cos^2(\pi x)$

5. Comparing the results obtained in the above exercises, what type of rectangles gives the best estimation for a fixed number of rectangles?

6. Evaluate the following indefinite integrals. Check that the resulting function is an actual primitive by direct derivation or simplifying to zero the difference between the derivative of the resulting function and the integrand function,

(a)

$$\int \frac{x}{x^3+1}\,dx$$

(b)

$$\int \frac{x}{x^5+1}\,dx$$

(c)

$$\int \frac{x^4-4x^3+2x^2-3x+1}{x^6+3x^4+3x^2+1}\,dx$$

(d)

$$\int \cos^5 x\,dx$$

(e)

$$\int \sin(3x)\cos(5x)\,dx$$

(f)

$$\int \frac{1}{1+\sin x+\cos x}\,dx$$

(g)

$$\int \frac{1}{x\sqrt{1+x^2}}\,dx$$

(h)

$$\int \frac{1}{x\sqrt{4+\ln^2 x}}\,dx$$

7. You want to paint a 1 unit sided square tile using two different colors. In order to separate the colors you plan to engrave two wiggle graphs joining the ends of one diagonal in such a way that the tangents to these wiggles at each end divide the corresponding angle into three equal parts. The region between the two wiggles is painted in one color and the outside region in the other one.

   (a) Find the relation between both color amounts if the wiggles are to be arcs of cubic polynomials.
   (b) Find fourth degree polynomials such that the three regions separated by the corresponding wiggles had the same amount of color.

8. Let $a > 0$ and

$$f(x) = e^{-ax} \sin x.$$

   (a) Plot a graph of f for $a = 0.1$.
   (b) Find the ratio of the height of two consecutive peaks.
   (c) Find the ratio between the area of two consecutive cycles.
   (d) Find the ratio between the volume of the solid generated by two consecutive cycles rotating around the x-axis.