CALCULUS
Lab Exercise Set 3

1 New Maple Commands

- **diff**$(f(x), x)$ derivative of an expression *f(x)* with respect to x
- **diff**$(f(x), x\$n)$ Derivative of order n with respect to x
- **D**(f) Derivative of a function/operator
- **(D@@n)**(f) n-order derivative of a function/operator
- **Diff**$(f(x), x)$ Symbol of the derivative of *f(x)* with respect to x
- **Diff**$(f(x), x)$ Symbol of the n-order derivative of *f(x)* with respect to x
- **animate**$(f(x, h), x = a..b, h = c..d)$ Animates a plot that depends on the parameter h
- **expand**$(expr)$ Expands an expression
- **normal**$(expr)$ Converts an expression into a ratio of two expressions
- **lhs**$(a = b)$ Returns the left hand side a of an equation
- **rhs**$(a = b)$ Returns the right hand side b of an equation
- **s**$[n]$ Extracts the n-th term of the sequence s
- **sum**$(f(n), n = 1..m)$ Calculates $f(1) + \cdots + f(m)$

2 Exercises

1. Geometric meaning of the derivative of a function $f(x)$ at a point $x = a$. Let $f(x) = x^2$ and $a = 1$. Answer the following questions:

   (a) Determine the slope and the equation of the secant line that crosses the graph of f at the points $(a, f(a))$ and $(a+h, f(a+h))$ as a function of the increment h of x.

   (b) Graph on the same set of axis the function f and the secant lines when h takes the values $1, 0.7, 0.4$ and 0.1.

   (c) Find the limit m of the slopes of the secant lines as h approaches 0.

   (d) Using your computer algebra system find the derivative of $f(x)$ at $x = a$ and check that it coincides with the value of m.

   (e) Find the equation of the line passing through the point $(a, f(a))$ and whose slope is m.This line is the tangent line to the graph of $f(x)$ at $x = a$. Graph the function f and the tangent line on the same set of axis.

2. Find the following derivatives

   (a) First derivative of
   $$\frac{\tan(x^2)}{x + \cos x}.$$

   (b) Second derivative of
   $$\sqrt[3]{2 - \sqrt[3]{2 - x}}.$$

   (c) Third derivative of
   $$x^{x^x}.$$

   (d) Fourth derivative of
   $$\sin^4(2x).$$

   (e) Fifth derivative of
   $$e^{-x^2}.$$

3. Find the general equation of fifth degree polynomial whose roots form a finite arithmetic sequence. Prove that the roots of its second derivative also form a finite arithmetic sequence.

4. Find a rational function

$$f(x) = \frac{ax^2 + bx + c}{x^2 + dx + e}.$$

and a number A such that the following conditions are met:

   (a) $f(x)$ has a pole at $x = 0$
   (b) $f(A) = A$.
   (c) $f'(A) = 0$.
   (d) $f(-3A) = -3A$.
   (e) $f''(-3A) = 0$.
   (f) $\lim_{x\to\infty} f(x) = 1$.

5. You are planning to make a closed rectangular box from a $W \times L$ piece of cardboard by cutting squares from the corners of one side and congruent rectangles from the other side and folding the sides up conveniently.

   (a) Find the dimensions of the box of largest volume that can be made this way and its corresponding volume $V_{\max}$. Express your result as a function of W and L and assume $W < L$.
   (b) Verify that from all pieces of cardboard of fixed unit area, $W \times L = 1$, the one that yields the maximum $V_{\max}$ is the square piece of dimensions $W = L = 1$.

6. Given the points $(1, 1)$, $(2, 1)$, $(3, 4)$, $(4, 3)$ and $(5, 5)$ find the best linear fit passing through $(0, 0)$, i.e. the line $y = mx$ that minimizes the sum of the squared errors

$$\sum_{i=1}^{5} (mx_i - y_i)^2.$$

7. A civil engineer is planning to build a bridge over a 1km wide river in the form of a polynomial. At one end the slope of the road is 7% and at the other side is 5%. Find a cubic polynomial whose graph joins both ends of the road smoothly. Would it be possible to build the bridge in the form of a quadratic polynomial?

8. (4.3.5.12) A plane is about to initiate its landing approach when it is at distance r and altitude h from the runway head. At that time the plane is flying at a constant speed u. Find a cubic polynomial that could be an acceptable approaching path. In order to eliminate jerky accelerations at the beginning and at the end of the landing approach, repeat the problem using a fifth degree polynomial.