CALCULUS
Lab Exercise Set 2

1 New Maple Commands

- @ Composition Operator
- **numer**(*expr*) Numerator of a rational expression
- **collect**($expr, x$) Collects powers of x in a polynomial
- **sort**($expr, x$) Sorts a polynomial by powers of x
- **coeffs**($expr, x$) Returns a list with the coefficients of a polynomial
- **solve**($\{eqn1, eqn2, ...\}, \{var1, var2, ...\}$) Solves a system of equations with several unknowns
- **plot**($\{expr1, expr2, ...\}, x = a..b$) Multiple plot
- **fsolve**($eqn, x, a..b$) Approximates numerically the root of equation *eqn* on the interval $[a, b]$
- **limit**($f(x), x = a$) limit of an expression $f(x)$ when x approaches a
- **limit**($f(x), x = a$,**right**) right-hand limit
- **limit**($f(x), x = a$,**left**) left-hand limit

2 Exercises

1. Let g be the function defined by

$$g(x) = \frac{ax+b}{cx+1}.$$

   Find a relation between a, b and c such that $g(g(x)) = x$ for every x.

2. In each of the following cases graph the given functions together on the same plot. Then, indicate what kind of transformation is needed to transform one into another.

   (a) $f(x) = x^2$ y $g(x) = -x^2$.
   (b) $f(x) = x^3$ y $g(x) = (-x)^3$.
   (c) $f(x) = x^3$, $g(x) = (x-1)^3$ y $h(x) = (x+1)^3$.
   (d) $f(x) = x^2$, $g(x) = x^2 - 1$ y $h(x) = x^2 + 1$.

3. In the following cases graph the function $f(x) = \sin(x)$ together with the given functions on the same plot. After observing these plots indicate how the variation of the parameters a, b and c affect the graph of $y = a\sin(bx+c)$.

   (a) $g(x) = \sin(x + \pi/6)$ and $h(x) = \sin(x + \pi/3)$.
   (b) $g(x) = \sin(2x)$ and $h(x) = \sin(3x)$.
   (c) $g(x) = 2\sin(x)$ and $h(x) = 3\sin(x)$.

4. Find the roots of the following polynomials. Try to find exact roots when possible, otherwise approximate them. Plot their graphs and check the roots on the graph.

   (a) $x^4 + x^3 - 7x^2 - x + 6$
   (b) $x^5 + x - 3$
   (c) $x^5 + 3x^4 + 2x + 10$
   (d) $x^5 - 8x^4 + x + 1$
   (e) $5x^5 - 15x^3 + 10x + 1$

5. In the following cases find the limit of $f(x)$ as x approaches a. Plot a graph of the function on an interval containing a and determine how close x must be to a so that the value of $f(x)$ is within 0.001 of the limit.

   (a) $f(x) = 1.7x^2 - 3x + 0.4$; $a = 0$
   (b) $f(x) = |x - 6|$; $a = 6$
   (c) $f(x) = \dfrac{0.002}{x+1}$; $a = -1$
   (d) $f(x) = \dfrac{\sin(x)}{x}$; $a = 0$
   (e) $f(x) = \sin(1/x)$; $a = 0$

6. Find the left and right-hand limits of the following functions when x approaches 0. ¿Does a limit exist? Graph the function and confirm your answer.

   (a) $f(x) = \dfrac{x(1+x)}{|x|}$.
   (b) $f(x) = \sqrt{|x|}$.
   (c) $f(x) = \dfrac{1}{1 - 2^{1/x}}$.
   (d) $f(x) = \dfrac{\sin(x + |x|)}{x^2}$.
   (e) $f(x) = \dfrac{1}{x(3^{1/x} + 1)}$.

7. According to Newton's law, an object that weighs 1 kilogram on the surface of the earth weighs approximately

$$p = \frac{40.96}{d^2}$$

   kilograms at a distance d, given in thousand kilometers, from the center of the earth.

   (a) At what distance from the center of the earth a 70 kilogram astronaut weighs less than 50 kilogram? and, less than 1 kilogram?

(b) Use the concept of limit at infinity to explain the term "zero gravity" that describes the zero apparent weight of the astronaut when his spacecraft is half way to the moon.

8. The estimated value v, in thousand of dollars, of an industrial machine t years after its purchase is given by the function

$$v(t) = 230 + \frac{325}{3^{0.02t}}.$$

The limit of of $v(t)$ as t approaches infinity is called the machine salvage value.

(a) Graph the function $v(t), t \geq 0$.

(b) What is the value of the machine at the time of purchase?

(c) What is its value after 1 year? and, after 10, 20, ... , 50 years?

(d) What is its salvage value?

(e) After how many years its price will be within 10 thousand dollars of the salvage value?