# CALCULUS
# Lab Exercice Set 1

## 1 Maple Commands

- **Arithmetic operators** +,-, *, /, ^ or **
- **simplify(***expr***)**; simplifies the expression enclosed in parentheses
- **evalf(***expr***)**; evaluates *expr*
- **var:=***expr*; assigns *expr* to the variable *var*
- **var:='***var***'**; unassigns the current value of the variable *var*
- **solve(***eqn***,{***var***})**; solves *eqn* for the unknown variable *var*
- **abs(***expr***)**; returns the absolute value of *expr*
- **sin, cos, tan, arctan, exp, ln**,...; names of some elementary functions
- **proc(x)** *expr* **end**; function of variable $x$ represented by the expression *expr*
- **x->***expr*; function of variable $x$ represented by the expression *expr*
- **unapply(***expr***,***x***)**; Converts the expression *expr* into a function of $x$
- **seq(**$f(i), i = 1..10$**)**;10 first terms of the sequence $f(n)$
- **array(**$[[a1, a2], [b1, b2], [c1, c2], [d1, d2]]$**)**; 4x2 table

- **plot**$(f(x), x = a..b)$; graphs an expression *expr* of $x$ on the interval $[a, b]$
- **plot**$(f, a..b)$; graphs a function $f$ on the interval $[a, b]$

# 2 Exercises

1. Calculate the following arithmetic expressions. Show the result in both symbolic and scientific notation
   (a) $(318 - 65)^8(17^5 - 73283) - 857713(3^{12})$
   (b) $\dfrac{\left(\frac{2}{5}\right)^3 - \frac{7}{3} + 183}{275 + 102^{14}}$
   (c) $\dfrac{1 + (3 - \sqrt{2})^2}{\sqrt[3]{4} + 16}$
   (d) $(2^{\frac{1}{3}} + 4^{\frac{1}{3}})^3 - 6(2^{\frac{1}{3}} + 4^{\frac{1}{3}})$
2. Calculate $\pi$ with 6, 10 and 100 digit precision.
3. Calculate the following expression with a precision of your choice $e^{\frac{1}{3}\pi\sqrt[3]{163}}$.
4. Plot the graph of the functions defined by the following expressions indicating their domain. Then evaluate each function at $x = -3, -1/2, 1, 2, \pi, 5$ as long as they belong to the function domain.
   (a) $f(x) = x^4 - 3x^2 + 5x - 2$
   (b) $f(x) = \dfrac{x}{1 + x^2}$
   (c) $f(x) = \sqrt{25 - x^2} + \sqrt{x^2 - 1}$
   (d) $f(x) = e^{-x^4} \cos x^3 \sin x^2$
   (e) $f(x) = \arctan(\ln|x^2 - 1|)$
5. The town council of a seaside town is planning to construct a port to carry goods from the town to a small offshore island. The distance from the island to the closest point on the shore is 1 mile and the distance from this point to the town 10 miles. The goods will be carried by truck from the town to the port at a speed of 50 miles per hour and by boat from the port to the island at a speed of 20 miles per hour.

(a) Find a mathematical model expressing the total time needed to carry the goods from the town to the island as a function of the distance from the town to the port.

(b) Evaluate this function at distances $d = 0, 1, 2, \ldots, 10$ miles and build a two entry table with the distances and the corresponding times.

(c) Plot a graph of the function and recommend the optimum location of the port to minimize the time from the town to the island.

6. In the following cases define the piecewise function $f$ by an algorithm and plot its graph

(a) $f(x) = \begin{cases} 1 & \text{if } x < 2 \\ -x + 4 & \text{if } x \geq 2 \end{cases}$

(b) $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$

(c) *Signum function* $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ +1 & \text{if } x > 0 \end{cases}$

(d) *Unit step function* $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

(e) *A pulse* $f(x) = \begin{cases} 1 & \text{if } 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$

7. Tax returns for married individuals are calculated using the following tax rate table

| **Taxable Income** | **Tax** |
|---|---|
| Less than \$16,700 | 10% of taxable income |
| From \$16,700 to \$67,900 | \$1,670 + 15% of taxable income over \$16,700 |
| From \$67,900 to \$137,050 | \$9,350 + 25% of taxable income over \$67,900 |
| From \$137,050 to \$208,850 | \$26,637.50 + 28% of taxable income over \$137,050 |
| From \$208,850 to \$372,950 | \$46,741.50 + 33% of taxable income over \$208,850 |
| Over \$372,950 | \$100,894.50 + 35% of taxable income over \$372,950 |

(a) Express the tax return $T$ as a function of the taxable income $x$.

(b) Build a two entry table showing the taxable income and the corresponding tax return for the following taxable incomes, $x = \$10,000 + 20,000 * n$, $n = 1, 2, ..., 20$.

(c) Plot a graph of $T(x)$ on the interval $0 \leq x \leq 400,000$.