CALCULUS

Midterm Exam
April 7, 2015
Duration: 45m

Exercise 1 *Find the values of x for which the geometric series*

$$\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{n-1}(x-2)^n$$

converges. Also, find the sum of the series for those values of x.

Solution Factoring out $x-2$ and doing $m=n-1$ the series can be written in the following form

$$\begin{aligned}\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{n-1}(x-2)^n &= (x-2)\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{n-1}(x-2)^{n-1}\\ &= (x-2)\sum_{m=0}^{\infty}\left(-\frac{1}{3}\right)^{m}(x-2)^{m}\\ &= (x-2)\sum_{m=0}^{\infty}\left(-\frac{x-2}{3}\right)^{m}.\end{aligned}$$

The series $\sum_{m=0}^{\infty}\left(-\frac{x-2}{3}\right)^m$ is a geometric series with ratio

$$r=-\frac{x-2}{3}.$$

This series converges as long as $|r|<1$, i.e. $|x-2|<3$ or $-1<x<5$.When convergent the sum of the series is

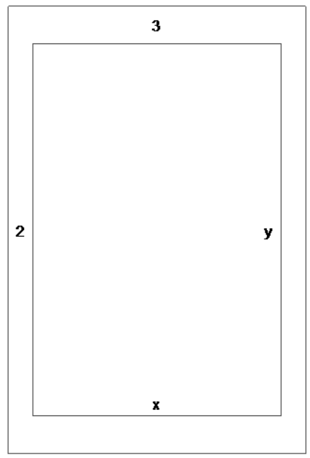
$$\begin{aligned}\sum_{m=0}^{\infty}\left(-\frac{x-2}{3}\right)^{m} &= \frac{1}{1-\left(-\frac{x-2}{3}\right)}\\ &= \frac{3}{1+x}.\end{aligned}$$

Therefore if $-1<x<5$ the given series is convergent and

$$\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{n-1}(x-2)^n=\frac{3(x-2)}{1+x}.$$

Exercise 2 *You are designing a rectangular poster with a blue border and a white center. The center is to contain 600 square inches of print. The border is to be 3 inches wide at top and bottom and 2 inches on each side. Find the most economical dimensions of the poster if the cost of the paper is 1 cent per square inch. What is the minimum cost?*

Solution



We have to minimize the total cost of the poster

$$C = (x+4)(y+6)$$

while keeping the area of the center fixed

$$xy = 600.$$

From this condition we obtain

$$y = \frac{600}{x}$$

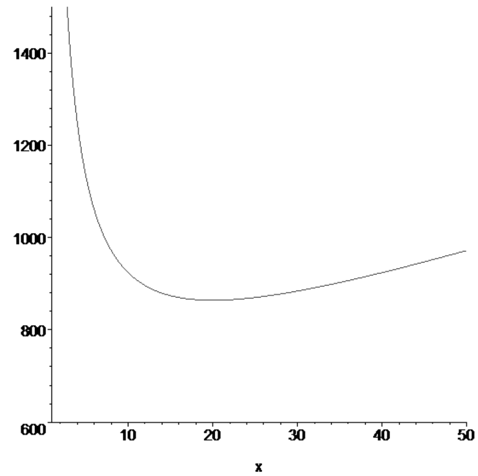
and

$$\begin{aligned} C &= (x+4)(\frac{600}{x}+6) \\ &= 624 + 6x + \frac{2400}{x}. \end{aligned}$$

Therefore, we have to find the minimum of this function in the region $x > 0$. The critical points are the solutions of the equation

$$C' = 6 - \frac{2400}{x^2} = 0$$

i.e. $x = 20$. Since there is only one critical point and $C \to +\infty$ when $x \to 0^+$ or $x \to +\infty$ we conclude that C reaches an absolute minimum at this point,



The minimum cost is $C(20) = 840$ cents.