CALCULUS

End-of-Term Exam
June 2, 2015
Duration: 45m

Exercise 1 *Find the tangent(s) to the curve*

$$\begin{aligned} x(t) &= t^2 + 2t + 1 \\ y(t) &= t^4 - 4t^2 \end{aligned}$$

at the point $(1, 0)$*. Do a plot of the curve and the tangent(s). (The plot counts for the lab grade)*

Solution First we need to find the values of t such that $x(t) = 1$ and $y(t) = 0$ simultaneously, i.e. the values of t such that

$$t^2 + 2t + 1 = 1$$

and

$$t^4 - 4t^2 = 0.$$

The roots of the first equation are $t = 0$ and $t = -2$. The roots of the second one are $t = 0, t = -2$ and $t = 2$.Therefore, the values of t that solve both equations simultaneously are $t = 0$ and $t = -2$.

Now we need to find the tangent vector at the point $(1, 0)$ for these two values of t. By differentiation we get

$$\begin{aligned} x'(t) &= 2t + 2 \\ y'(t) &= 4t^3 - 8t. \end{aligned}$$

and the tangent vectors are

$$\begin{aligned} x'(0) &= 2 \\ y'(0) &= 0 \end{aligned}$$

and

$$\begin{aligned} x'(-2) &= -2 \\ y'(-2) &= -16. \end{aligned}$$

Therefore the equations of the tangent lines at the point $(1, 0)$ are

$$\frac{x-1}{2} = \frac{y-0}{0}$$

and

$$\frac{x-1}{-2} = \frac{y-0}{-16}$$

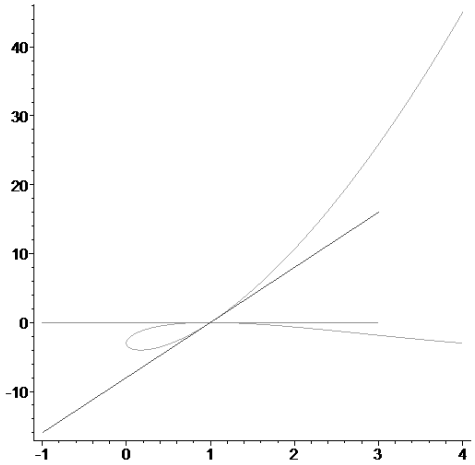
or

$$y = 0$$

and

$$y = 8x - 8.$$

Plot

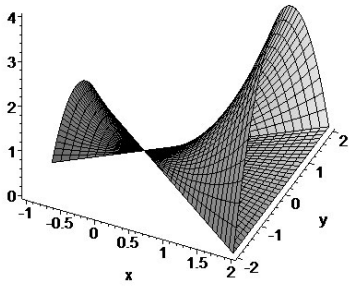


Exercise 2 *Evaluate the double integral*

$$\iint_D \left(x^2 - y^2\right)\, dA$$

where D is the region enclosed by the lines $y = x$ and $y = -x$ between $x = -1$ and $x = 2$. Do a $3D$ plot of the surface represented by the function over the domain D.(The plot counts for the lab grade)

Solution Plot of the function over the given domain



To calculate the integral we need to break it into two integrals, one between $x = -1$ and $x = 0$ and another one between $x = 0$ and $x = 2$.

$$\begin{aligned}
\iint_D \left(x^2 - y^2\right)\, dA &= \int_{-1}^{0} \left(\int_{x}^{-x} \left(x^2 - y^2\right)\, dy \right) dx + \int_{0}^{2} \left(\int_{-x}^{x} \left(x^2 - y^2\right)\, dy \right) dx \\
&= \int_{-1}^{0} \left(-\frac{4}{3}x^3 \right) dx + \int_{0}^{2} \left(\frac{4}{3}x^3 \right) dx \\
&= \frac{1}{3} + \frac{16}{3} \\
&= \frac{17}{3}.
\end{aligned}$$